

# Triangle Congruence

by Dalide Pontoni

## LESSON 5

Today we want to check our ability in proving theorems and review all the basic concepts that we have studied.

### PLAN

- First we **check** the homework.
- Then we **practice!**

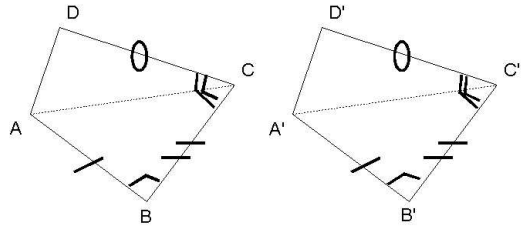
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## EXERCISE 5.1

Given the following exercise, put all the different steps of its proof in the correct order. (Team work).

Let  $ABCD$  and  $A'B'C'D'$  be two quadrilaterals such that  $AB \cong A'B'$ ,  $BC \cong B'C'$ ,  $CD \cong C'D'$ ,  $\angle ABC \cong \angle A'B'C'$  and  $\angle BCD \cong \angle B'C'D'$ . Trace the two diagonals  $AC$  and  $A'C'$ . Prove that the quadrilaterals are congruent, that is to say (=cioè) their corresponding sides and angles are congruent.



1) **Proof:**

2) **Hypotheses:**

3) **Thesis:**

4) **Drawing**

5) Now consider the triangles  $ADC$  and  $A'D'C'$ . They have:

6) In particular,  $AC \cong A'C'$ ,  $\angle BAC \cong \angle B'A'C'$  and  $\angle ACB \cong \angle A'C'B'$ .

7) Therefore they are congruent by the SAS criterion.

8)  $AC \cong A'C'$  as shown above;

9)  $BC \cong B'C'$  by hypothesis;

10)  $AB \cong A'B'$ ,  $BC \cong B'C'$  and  $CD \cong C'D'$ ;

11) Therefore they are congruent by the SAS criterion.

12)  $ABCD \cong A'B'C'D'$ .

13)  $\angle ACD \cong \angle A'C'D'$  because they are the difference of congruent angles;

14)  $AB \cong A'B'$  by hypothesis;

15)  $CD \cong C'D'$  by hypothesis;

16) In particular,  $AD \cong A'D'$ ,  $\angle ADC \cong \angle A'D'C'$  and  $\angle DAC \cong \angle D'A'C'$ .

17) Since also  $\angle DAB \cong \angle D'A'B'$  because they are the sum of congruent angles, the two quadrilaterals have congruent corresponding sides and angles. Hence they are congruent.

18) Consider the two triangles  $ABC$  and  $A'B'C'$ . They have:

19)  $\angle ABC \cong \angle A'B'C'$  and  $\angle BCD \cong \angle B'C'D'$ .

20)  $\angle ABC \cong \angle A'B'C'$  by hypothesis.

# Triangle Congruence

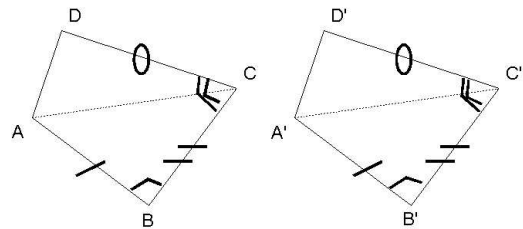
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## EXERCISE 5.1 (Solution)

Given the following exercise, put all the different steps of its proof in the correct order.

Let  $ABCD$  and  $A'B'C'D'$  be two quadrilaterals such that  $AB \cong A'B'$ ,  $BC \cong B'C'$ ,  $CD \cong C'D'$ ,  $\angle ABC \cong \angle A'B'C'$  and  $\angle BCD \cong \angle B'C'D'$ . Trace the two diagonals  $AC$  and  $A'C'$ . Prove that the quadrilaterals are congruent, that is to say (=cioè) their corresponding sides and angles are congruent.

### 4) Drawing



### 2) Hypotheses:

- 10)  $AB \cong A'B'$ ,  $BC \cong B'C'$  and  $CD \cong C'D'$ ;  
 19)  $\angle ABC \cong \angle A'B'C'$  and  $\angle BCD \cong \angle B'C'D'$ .

### 3) Thesis:

- 12)  $ABCD \cong A'B'C'D'$ .

### 1) Proof:

- 18) Consider the two triangles  $ABC$  and  $A'B'C'$ . They have:  
 14)  $AB \cong A'B'$  by hypothesis;  
 9)  $BC \cong B'C'$  by hypothesis;  
 20)  $\angle ABC \cong \angle A'B'C'$  by hypothesis.  
 7) Therefore they are congruent by the SAS criterion.  
 6) In particular,  $AC \cong A'C'$ ,  $\angle BAC \cong \angle B'A'C'$  and  $\angle ACB \cong \angle A'C'B'$ .  
 5) Now consider the triangles  $ADC$  and  $A'D'C'$ . They have:  
 8)  $AC \cong A'C'$  as shown above;  
 15)  $CD \cong C'D'$  by hypothesis;  
 13)  $\angle ACD \cong \angle A'C'D'$  because they are the difference of congruent angles.  
 11) Therefore they are congruent by the SAS criterion.  
 16) In particular,  $AD \cong A'D'$ ,  $\angle ADC \cong \angle A'D'C'$  and  $\angle DAC \cong \angle D'A'C'$ .  
 17) Since also  $\angle DAB \cong \angle D'A'B'$  because they are the sum of congruent angles, the two quadrilaterals have congruent corresponding sides and angles.  
 Hence they are congruent.

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## EXERCISE 5.2

Look at the following drawings, read the hypotheses and theses and write the statement of the corresponding exercise. Work in team!

**a) Hypotheses:**

$AB \cong AC$

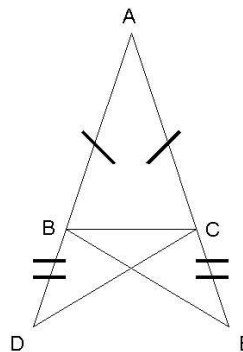
$BD \cong CE$

A, B, D collinear points

A, C, E collinear points

**Thesis:**

$CD \cong BE$



**Statement:**

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**b) Hypotheses:**

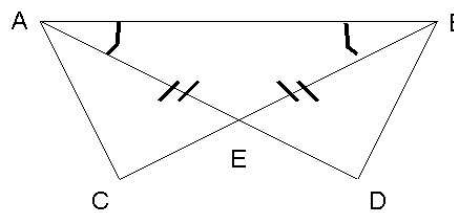
$AD \cong BC$

$\angle BAD \cong \angle ABC$

$AD \cap BC = \{E\}$

**Thesis:**

$EC \cong ED$



**Statement:**

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## EXERCISE 5.3

Look at the drawing and read the following hypotheses and theses. For each list of hypotheses decide if they contain enough information in order to prove the corresponding thesis. (Team work).

### 1. Hypotheses:

$$AD \cong DB$$

$$AE \cong CB$$

### Thesis:

$$ADE \cong BDC$$

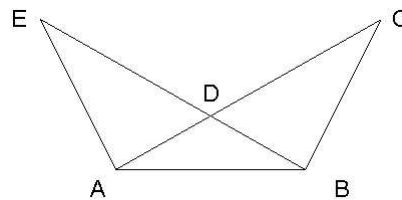
### 2. Hypotheses:

$$\angle DAB \cong \angle DBA$$

$$EB \cong AC$$

### Thesis:

$$ADE \cong BDC$$



### 3. Hypotheses:

$$ABE \cong BAC$$

### Thesis:

$$AD \cong DB$$