

Triangle Congruence

by Dalide Pontoni

LESSON 4

Today we want to complete our study by learning the third "trick" used to decide about the congruence of triangles.

PLAN

- First we **check** the homework and **review** some properties of the isosceles triangles.
- Then we learn the **third criterion** of congruence.
- Finally we **prove** some theorems involving the third criterion.

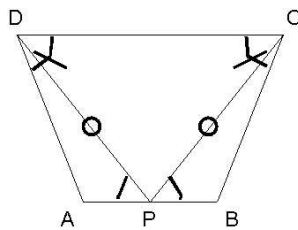
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EXERCISE 4.1

WORK IN PAIRS!

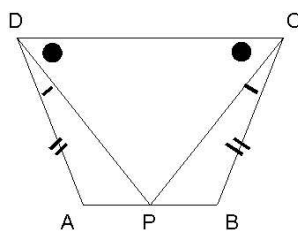
Look at the drawing and complete the sentences.



- a) The triangle PCD is isosceles because.....
.....
- b) $\angle PDC \cong \angle PCD$ because.....
.....
- c) $\angle ADP \cong \angle BCP$ because
.....
- d) The triangle APD is congruent to the triangle BPC by

EXERCISE 4.2 (Homework)

Look at the drawing and complete the sentences.



- a) The triangle PCD is isosceles because.....
- b) $PD \cong PC$ because
- c) The triangle APD is congruent to the triangle BPC by
- d) $\angle PAD \cong \angle PBC$ because.....

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THE THIRD CONGRUENCE CRITERION : SSS = Side – Side - Side

If the **three sides** of a triangle are congruent to the corresponding sides of a second triangle, then the triangles are congruent.

Hypotheses:

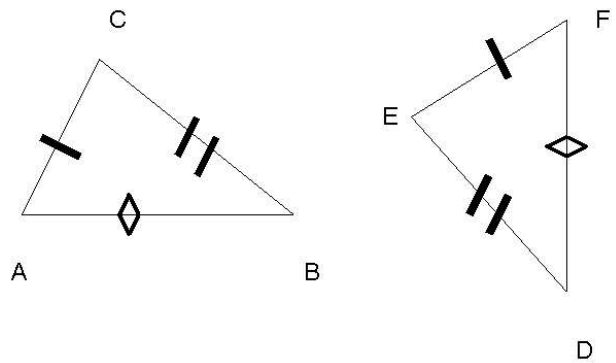
$$AC \cong FE$$

$$AB \cong FD$$

$$BC \cong DE$$

Thesis:

$$ACB \cong FED$$



QUESTION:

Can it exist also an AAA criterion of congruence? Why?

Discuss (in Italian) with your desk mate and try to answer the question (in English).
You can also draw some pictures and make some examples.

.....
.....
.....

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EXERCISE 4.3

Let ABC and $A'B'C'$ be two triangles such that $AB \cong A'B'$, $AC \cong A'C'$. Denote by CM and $C'M'$, respectively, the medians passing through the vertex angles $\angle ACB$ and $\angle A'C'B'$. Prove that if also CM and $C'M'$ are congruent, then ABC and $A'B'C'$ are congruent.

Hypotheses:

.....

Drawing

Thesis:

.....

Proof:

The triangles ACM and $A'C'M'$ are such that:

- $AM \cong$ by
- $AC \cong$ by
- $CM \cong$ by

Hence they are by the criterion.

In particular, $\angle CAM \cong$

Consider now the triangles ABC and $A'B'C'$. They have:

- $AB \cong$ by.....
- $AC \cong$ by.....
- $\angle CAB \cong$ by the previous proof.

Therefore, they are by the criterion. □

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EXERCISE 4.4 (Homework)

a) The triangles ABC and DEF are equilateral and $AB \cong DE$. Are they congruent?

Why?.....
.....
.....
.....

b) The triangles ABC and DEF are both isosceles. Their bases AB and DE, respectively, are congruent and also the sides AC and DF are congruent. Are the two triangles congruent?

Why?.....
.....
.....
.....

EXERCISE 4.5 (Homework)

Let AB be the base of the isosceles triangle ABC. Draw the equilateral triangles BCD and ACE with E and D exterior points with respect to ABC. Denote by F the intersection point of AD and BE. Prove that:

- a) $AD \cong BE$;
- b) CF is the bisector of angle $\angle ACB$.

(Hints: a) Consider the triangles ABD and ABE.
b) What kind of triangle is ABF? Why?
Are the triangles FCA and FCB congruent? Why?)