

# Triangle Congruence

by Dalide Pontoni

## LESSON 3

Today we want to improve our abilities in proving theorems!

### PLAN

- First we **check** the homework.
- Then we **practice** and **learn** some properties of the isosceles triangle.

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## EXERCISE 3.1

Given the following theorem, put all the different steps of its proof in the correct order.

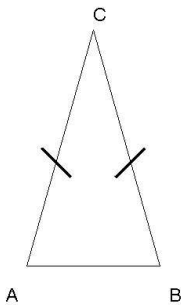
### THEOREM

If the triangle ABC is **isosceles** on the base AB, then its **base angles** (=angoli alla base) are congruent.

#### 1) Proof:

2)  $AC \cong BC$  by hypothesis

#### 3) Drawing



4)  $\angle CAB \cong \angle ABC$

5) Let CH be the angle bisector of  $\angle ACB$ .

#### 6) Thesis:

7) Hence they are congruent by the SAS criterion.

8) In particular,  $\angle CAH \cong \angle CBH$ .

#### 9) Hypothesis:

10) CH in common

11) Consider the triangles AHC and BHC. They have:

12)  $AC \cong BC$

13)  $\angle ACH \cong \angle BCH$  because CH is the bisector.

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## EXERCISE 3.1 (Solution)

Given the following theorem, put all the different steps of its proof in the correct order.

### THEOREM

If the triangle ABC is **isosceles** on the base AB, then its **base angles** (=angoli alla base) are congruent.

#### Hypotheses:

$$AC \cong BC$$

#### Thesis:

$$\angle CAB \cong \angle ABC$$

#### Proof:

Let CH be the angle bisector of  $\angle ACB$ .

Consider the triangles AHC and BHC. They have:

$$AC \cong BC \quad \text{by hypothesis}$$

$$CH \quad \text{in common}$$

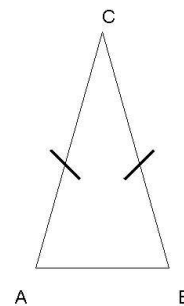
$$\angle ACH \cong \angle BCH \quad \text{because CH is the bisector.}$$

Hence they are congruent by the SAS criterion.

In particular,  $\angle CAH \cong \angle CBH$ .

□

#### Drawing



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## EXERCISE 3.2 (Homework)

The proof of the previous theorem justifies the following sentences. Why?

- 1) The angle bisector CH is also the **median** (= mediana) passing through the **midpoint** (= punto medio) of the base AB.
- 2) The angle bisector CH is perpendicular to the base AB, hence it is also an **altitude** (= altezza).

## EXERCISE 3.3 (Homework)

Complete the proof of the converse of the theorem 3.1, after sketching the suggested drawing.

### CONVERSE THEOREM

If the triangle ABC has **two congruent angles**, then it is an **isosceles triangle**.

**Hypothesis:**

.....

**Drawing**

**Thesis:**

.....

**Suggested drawing** (it helps in finding congruent triangles!)

Sketch a triangle ABC (you can already draw an isosceles triangle with base AB). Trace the angle bisectors AP and BQ of angles  $\angle BAC$  and  $\angle ABC$ , respectively.

**Proof:**

1. Let us consider the triangles ABQ and ABP. They have:

- AB in common
- ..... by .....
- ..... because .....

Hence they are ..... by .....

In particular, AP ..... and  $\angle AQB$  .....

2. Let us consider the triangles APC and BQC. They have:

- AP ..... as shown above
- $\angle CAP$  ..... because .....
- $\angle CPA$  ..... because .....

Therefore they are ..... by ..... In particular, .....  $\square$