

Triangle Congruence

by Dalide Pontoni

LESSON 2

Today we want to practice a little, hence we will prove some more theorems by means of the SAS criterion.

We also want to improve (= migliorare) our abilities by learning a second "trick" which is very useful in order to decide about the congruence of triangles.

PLAN

- First we **check** the homework.
- Then we **prove** some theorems involving the SAS criterion.
- We learn the **second criterion** of congruence.
- Finally we **prove** some theorems involving the second criterion.

Triangle Congruence

by Dalide Pontoni

EXERCISE 2.2 (Homework)

Let C be a point on the angle bisector b of the convex angle $\angle XOY$. Let A and B be two points on the angle sides OX and OY , respectively, such that $OA \cong OB$.
Then the triangles BOC and AOC are congruent.

Hypotheses:

- 1).....
- 2).....
- 3).....
- 4).....
- 5).....

Drawing

Thesis:

.....

Proof:

.....
.....
.....
.....
.....
.....
.....

□

Triangle Congruence

by Dalide Pontoni

THE SECOND CONGRUENCE CRITERION : ASA = Angle – Side - Angle

If **two angles** in one triangle are congruent to the corresponding angles of a second triangle, and also the **included sides** (=lati compresi) are congruent, then the triangles are congruent.

We can also say:

If two triangles have **two angles** and the **included sides** congruent, respectively, then they are congruent.

Hypotheses:

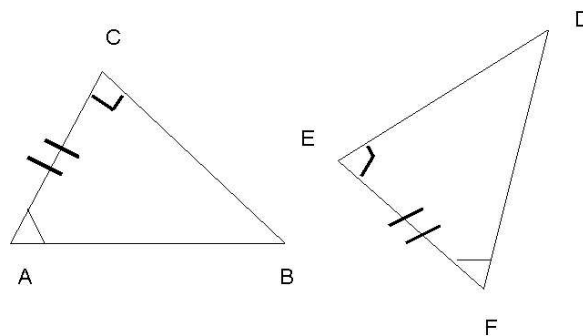
$$\angle CAB \cong \angle EFD$$

$$AC \cong FE$$

$$\angle ACB \cong \angle FED$$

Thesis:

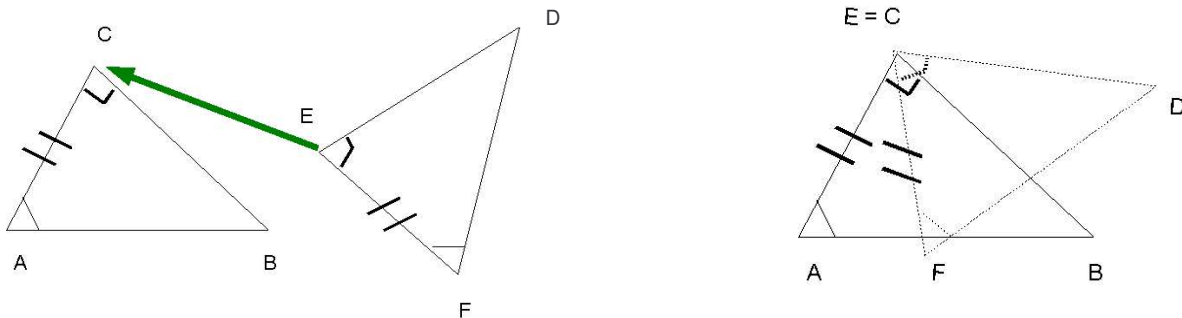
$$ACB \cong FED$$



Triangle Congruence

by Dalide Pontoni

WHY DOES IT WORK?



Thanks to an isometry, we can put vertex E on top of vertex C. Since $AC \cong FE$, we can rotate the triangle FED so that also vertices A and F coincide.

We notice that:

- since $\angle CAB \cong \angle EFD$, the side AB **lies** (to lie, lay, lain, lying = giacere, mettere - intransitivo) **on** the ray containing the side FD;
- since $\angle ACB \cong \angle FED$, necessarily the side BC **lies on** the ray containing the side DE.

Hence, also vertices B and D must coincide and the two triangles are congruent!

Triangle Congruence

by Dalide Pontoni

APPLICATION

EXERCISE 2.3

The bisector of angle $\angle CAB$ of the triangle ABC intersects the side BC at the point D . Trace (to trace = tracciare una linea su una superficie) the ray with origin D intersecting AB at the point E so that $\angle EDA \cong \angle ADC$. Prove that CD and DE are congruent.

Hypotheses:

.....

Drawing

Thesis:

.....

Proof:

Segments CD and DE , respectively, are sides of the triangles.....
 The two triangles have:

- $\angle CAD \cong$ by
- $\angle ADC \cong$ by
- side in common

Hence,by criterion.

In particular, the corresponding sides and are congruent.



Triangle Congruence

by Dalide Pontoni

EXERCISE 2.4 (Homework)

Triangles ABC and $A'B'C'$ are congruent. Let AD and $A'D'$, respectively, be the angle bisectors of the corresponding angles $\angle BAC$ and $\angle B'A'C'$. Show that AD is congruent to $A'D'$.

Hypotheses:

.....

Drawing

Thesis:

.....

Proof:

Let us consider the triangles They have:

- $AB \cong$ by
- $\angle ABC \cong$ by
- $\angle DAB \cong$ because.....

Therefore \cong by criterion.

In particular, $AD \cong$

□

EXERCISE 2.5 (Homework)

Let ABC and $A'B'C'$ be two triangles satisfying the following properties: $BC \cong B'C'$ and $\angle ABC \cong \angle A'B'C'$. Choose a point P and a point P' on the sides BC and $B'C'$, respectively, such that $\angle CPA \cong \angle C'P'A'$, $CP \cong C'P'$. Prove that $AC \cong A'C'$.

(Hints: 1) Are the angles $\angle APB$ and $\angle A'P'B'$ congruent? Why?

2) Maybe you should use both SAS and ASA criteria to prove the final thesis.)

EXERCISE 2.6 (Homework)

ABC and $A'B'C'$ are two triangles such that $\angle CAB \cong \angle C'A'B'$, $\angle ABC \cong \angle A'B'C'$ and $AB \cong A'B'$. Let M and M' be the midpoints of BC and $B'C'$, respectively. Analogously, let N and N' be the midpoints of AC and $A'C'$, respectively. Prove that $MN \cong M'N'$.

(Hints: 1) Are ABC and $A'C'B'$ congruent triangles? Why?

2) Maybe you should use both SAS and ASA criteria on different triangles.)