Triangle Congruence

by Dalide Pontoni

LESSON 1

We want to understand what it means that two triangles are **congruent**.
Moreover, we want to find out if there are some **criteria** (criterion/criteria=criterio/criteri) that allow us to decide about the congruence of two triangles.

PLAN

- First we **give the definition** of congruent triangles.
- Then we **learn** a “trick” to decide if two triangles are congruent or not, the so-called first congruence criterion.
- Finally we **try to prove** some theorems by means of (= per mezzo di) the criterion itself.

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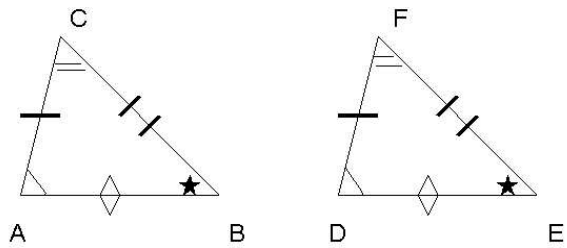
DEFINITION

We say that the **triangle ABC is congruent to the triangle DEF** if:

- $AB \cong DE$
- $BC \cong EF$
- $CA \cong FD$
- $\angle BAC \cong \angle EDF$
- $\angle ABC \cong \angle DEF$
- $\angle BCA \cong \angle EFD$



REMARK (= osservazione)

Note that congruent sides have the same length and congruent angles have the same amplitude, by the definition of congruence as an equivalence relation.

VERY IMPORTANT REMARK

The notation is fundamental! It includes information about which vertices and which sides **correspond**.
If we write $ABC \cong DEF$, then we mean that: side AB corresponds to side DE and they are congruent; side BC corresponds to side EF and they are congruent; …; the angle $\angle CAB$ corresponds to the angle $\angle FDE$ and they are congruent and so on.

Hence the notation $ABC \cong DEF$ **is not the same as** $ABC \cong FED$!

Once the order is set up (to set up = fissare, fondare - irregular verb) properly at the beginning, it is easy to read off all six congruences.

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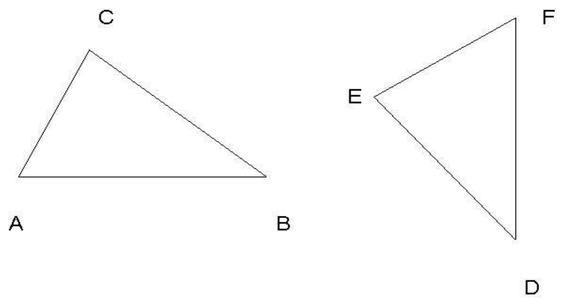
EXERCISE 1.1

1) Look at the following drawings. The two triangles are congruent. Write the pairs of the corresponding congruent sides and angles and then choose the correct notation of congruence among the list above.

a) ABC ≅ DEF
b) ABC ≅ EFD
c) BCA ≅ DEF
d) BCA ≅ EFD
e) CAB ≅ FDE
f) CAB ≅ FED



EXERCISE 1.2 (Homework)

True or false?

a) If two triangles are congruent, then they are both isosceles or both equilateral………………T F
b) All equilateral triangles are congruent……………………………………………………...T F
c) If an equilateral and a scalene triangle have the same extension, then they are congruent …...T F
d) If with an isometry we can put a triangle on the top of another one so that the two triangles coincide point by point, then they are congruent ……………………………..………..…...T F
e) If two triangles are congruent, then their corresponding elements are pairs of sides and pairs of vertices……………………………………………………………………………………T F

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THE FIRST CONGRUENCE CRITERION : SAS = Side – Angle - Side

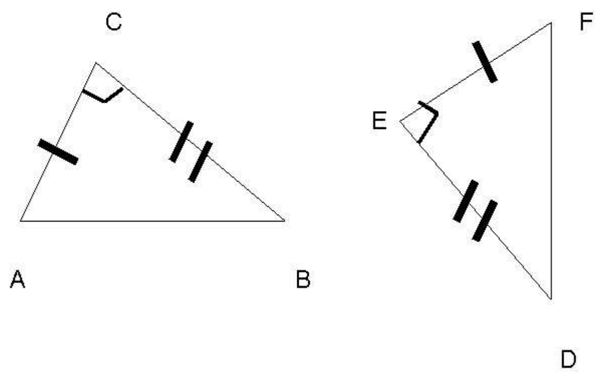
If **two sides** in one triangle are congruent to the corresponding sides of a second triangle, and also the **included angles** (=angoli compresi) are congruent, then the triangles are congruent.

We can also say:

If two triangles have **two sides** and the **included angles** congruent, respectively, then they are congruent.

Hypotheses:
$AC \cong EF$
$\angle ACB \cong \angle FED$
$BC \cong DE$

Thesis:
$ACB \cong FED$



VERY IMPORTANT REMARK

In general, the SSA criterion is not valid!
Verify this sentence by drawing two triangles ABC and DEF such that: $AB \cong DE$, $BC \cong EF$ and $\angle BAC \cong \angle EDF$, (you should use a ruler, a compasses and a protractor).

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APPLICATIONS

EXERCISE 1.3

Draw a triangle ABC. Extend (to extend = prolungare) AB by a segment BE ≅ AB and CB by a segment BF ≅ CB. Join (to join = unire, congiungere) E to F. Prove that AC ≅ EF.

Hypotheses: **Drawing**

1)…………………
2)…………………
3)…………………

Thesis:

……………………

Proof:

The triangles ABC and EBF are such that:

- AB ≅ …… by hypothesis ……;
- CB ≅ …… by ………………..;
- ∠ABC ≅ ∠FBE because they are …………………… angles.

Hence they are ……………. by SAS.

In particular: AC ≅……… □

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EXERCISE 1.4 (Homework)

Let AB be a line segment and M its midpoint. Let *r* be any (=qualsiasi) straight line passing through M. On the line *r* , at opposite sides with respect to AB, draw two congruent segments ME and MF. Join A to F, and B to E. Prove that the triangles AMF and BME are congruent.

Hypothesis: **Drawing**

1)…………………….
2)…………………….
3)…………………….
4)…………………….

Thesis:

……………………

Proof:

The triangles AMF and BME are such that:

- AM ≅…………….. by ………………………
- ME ≅ …………….. by ………………………
- ∠ AMF ≅ …………... because ……………………………………………………

Hence they are ………………………. by ………………………. □

EXERCISE 1.5 (Homework)

Let M be the midpoint of the side BC of the triangle ABC. Extend AM by a segment ME ≅ AM and prove that AC ≅ BE.

Hypotheses: **Drawing**

1)………………….
2)………………….
3)………………….

Thesis:

…………………….

Proof:

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……
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……
……
…… □