

TRIANGLE CONGRUENCE: FINAL TEST - CLASS 1° B - 19th March 2009

EXERCISE 1. Let AB be the base of the isosceles triangle ABC . Choose a point E and a point F on the sides AC and BC , respectively, such that $CE \cong CF$. Join E with B and A with F ; denote with D the intersection point of the segments BE and AF . Prove that ABD is an isosceles triangle.

Hypotheses:

Thesis:

Drawing

.....

Proof:

Consider the triangles ACF and They are such that:

AC because

CE by

They have the angle \widehat{ACB}

Hence they are by

In particular, $\widehat{CAF} \cong$

Can we say that \widehat{CAB} and \widehat{CBA} are congruent?

Give the reason:

We can now say that the angles \widehat{BAD} and are

because they are of congruent angles.

Finally, we can apply the on isosceles triangles to conclude

that is

EXERCISE 2. Look at the following drawing, read the hypotheses and thesis and write the statement of the corresponding theorem.

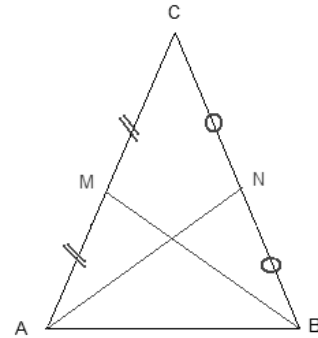
Hypotheses:

$AC \cong BC$
 $AM \cong CM$
 $CN \cong BN$

Thesis:

$BM \cong AN$

Drawing



STATEMENT:

EXERCISE 3. Prove the statement of exercise 2 (you do **not need** to write the hypotheses, the thesis and to sketch the drawing again).

EXERCISE 4. Given the following statement and proof, do the drawing and put all the steps in the correct order.

Let ABC be an isosceles triangle with base AB . Extend AB on both sides by two congruent segments DA and BE . Choose a point P and a point Q on the sides AC and BC , respectively, such that $AP \cong BQ$. Prove that $PE \cong DQ$.

1. **Proof**
2. $\widehat{CAB} \cong \widehat{ABC}$ because they are base angles
3. Observe that $DB = DA + AB$ and $AE = AB + BE$ by definition of sum of segments
4. **Drawing**
5. In particular, $PE \cong DQ$
6. ABC is an isosceles triangle on the base AB
7. BDQ and AEP are congruent by SAS (this follows from steps)
8. $AP \cong BQ$
9. Consider the triangles BDQ and AEP
10. $DA \cong BE$
11. So $DB \cong AE$ (in fact $DA \cong BE$ implies $DA + AB \cong AB + BE$)
12. **Thesis**
13. **Hypotheses**
14. We have $AP \cong BQ$ by hypothesis
15. $PE \cong DQ$

EXERCISE 5. (**) Consider the statement of exercise 4. Prove also that

- (a) if M is the intersection point of the segments DQ and PE , then DME is an isosceles triangle
- (b) M belongs to the angle bisector of \widehat{C}