EXERCISE SHEET CLIL MODULE - Relations and Functions CLASS 1° B - 01/23/2009

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EXERCISE 1. Consider the following functions: $f : \mathbb{N} \to \mathbb{N} \quad x \mapsto 2x+3$ and $g : \mathbb{N} \to \mathbb{N} \quad x \mapsto 2x^2+1$. Which of the following is the composite function $g \circ f$?

A	$x \mapsto 8x^2 + 24x + 19$
B	$x \mapsto 4x^2 + 5$
C	$x \mapsto (2x + 3)(2x^2 + 1)$
D	$x \mapsto 2x^2 + 4$
E	none of the preceding formulas

EXERCISE 2. Consider the function $f : \mathbb{Z} \to \mathbb{Z} \quad x \mapsto 2x + 1$. Which of the following are true sentences?

A	f is injective
B	f is invertible
C	the codomain is the set of odd numbers
D	$f(0) = 0$
E	$(f \circ f)(x) = 4x + 2$

EXERCISE 3. Given the relation $\mathcal{R} = \{(a, a), (a, b), (a, c), (c, a), (c, b), (c, c), (d, d)\}$ we can say that:

A	it is antireflexive
B	reflexive and symmetric
C	it is not reflexive and not symmetric
D	it is symmetric only
E	it is reflexive only

EXERCISE 4. Which is the domain of the function f defined by

$$x \mapsto \frac{1}{x} + \frac{x+1}{2x-6}$$

A	$A = \{x \in \mathbb{Q} \mid x \neq 0\}$
B	$\mathbb{Q}$
C	$A = \{x \in \mathbb{Q} \mid x \neq 0 \wedge x \neq 3\}$
D	$A = \{x \in \mathbb{Q} \mid x \neq 3\}$
E	$A = \{x \in \mathbb{Q} \mid x \neq 0 \wedge x \neq -1\}$

EXERCISE 5. Given the functions $f : A \to \mathbb{Z}$ defined by $x \mapsto |x| + 3$ and $g : \mathbb{Z} \to \mathbb{Z}$ defined by $x \mapsto x^2$ where $A = \{x \in \mathbb{Z} \mid -3 \leq x \leq 3\}$, tell which of the following are true sentences.

A	f is injective
B	the codomain of f contains 4 elements
C	the range of $g \circ f$ contains only perfect squares
D	$(f \circ g)(1) = (g \circ f)(1)$
E	none of the preceding

EXERCISE 6. Consider the following relation defined in the set of natural numbers $\mathbb{N}$:
$\mathcal{R} : x$ *has at least one digit in common with* y.
Which of the following sentences are true?

A	$\mathcal{R}$ is reflexive and symmetric
B	$\mathcal{R}$ is an equivalence relation
C	$\mathcal{R}$ is reflexive and transitive
D	$\mathcal{R}$ is reflexive and antisymmetric
E	none of the preceding

EXERCISE 7. Consider the equivalence relation $\mathcal{R}$ defined on $A = \{x \in \mathbb{N} \mid 1 \leq x \leq 5\}$ by the rule $x\mathcal{R}y$ if x and y give the same remainder when divided by 7. Which is the cardinality of the quotient set?

A	5
B	6
C	7
D	8
E	infinite

EXERCISE 8. Consider the set $A = \{x \in \mathbb{N} \mid 3 \leq x \leq 8\}$ and the relation $\mathcal{R} : x + y$ *is an even number* defined on A. Which of the following are true sentences?

A	$\mathcal{R}$ is reflexive and antisymmetric
B	$\mathcal{R}$ is an equivalence relation
C	the inverse relation $\mathcal{R}^{-1}$ is equal to $\mathcal{R}$
D	$\mathcal{R}$ is also a function
E	for every $a \in A$ there are exactly 2 elements $b \in A$ such that $a\mathcal{R}b$ with $b \neq a$

EXERCISE 9. Consider the following relations $\mathcal{R}_1$ and $\mathcal{R}_2$ defined in $A \times B$ where $A = \{x, y, z, w, u\}$ and $B = \{1, 3, 5, 7\}$:
$\mathcal{R}_1 = \{(x, 1), (y, 7), (z, 3), (w, 5), (u, 7)\}$
$\mathcal{R}_2 = \{(x, 1), (y, 3), (z, 5), (y, 7)\}$
Which of the following are true sentences?

A	$\mathcal{R}_1$ and $\mathcal{R}_2$ are both functions
B	$\mathcal{R}_1$ is surjective but not injective
C	$\mathcal{R}_2$ is injective
D	$\mathcal{R}_1 \cap \mathcal{R}_2$ is a function
E	it is not possible to define an injective function from A to B

EXERCISE 10. Given the function $f : \mathbb{Q} \to \mathbb{Q}$ defined by $x \mapsto 3x + 1$ what is the value $f^{-1}(0)$?

A	-1
B	0
C	$\frac{1}{3}$
D	$-\frac{1}{3}$
E	none of the preceding

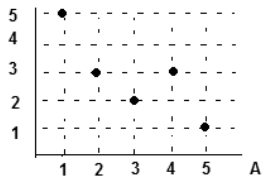
EXERCISE 11. Given the function $f = \{(1, b), (3, d), (2, b), (4, b)\}$ complete the following relations

A	$f(2) = \ldots$
B	the pre-image of b is $\ldots$
C	the domain of f is $\ldots$
D	the set of all images of f is $\ldots$
E	$f(\ldots) = d$

EXERCISE 12. Given the function $f : \mathbb{Q} \to \mathbb{Q}$ defined by $x \mapsto x^2 + 1$, complete the following relations:

A	$f(-2) = \ldots$
B	$f(0) = \ldots$
C	$f(\ldots) = 1$
D	$f(\ldots) = 2$
E	$f(\ldots) = 5$

EXERCISE 13. Consider the relation $\mathcal{R}$ represented by the graph below and tell which of the following sentences are true.



A	$\mathcal{R}$ is a surjective function
B	$\mathcal{R}$ is an injective function
C	$\mathcal{R}$ is not a function
D	the image of 1 is greater than the image of 3
E	$\mathcal{R}$ is an invertible function

EXERCISE 14. We are given the set $A = \{2, h, 4, k, 12\}$ and the function f defined by $x \mapsto x^2 + 1$. If the range of f is the set $B = \{5, 10, 17, 82, 145\}$, which must be the values of h and k in $\mathbb{N}$ such that A becomes the domain of f?

A	5 and 9
B	3 and 9
C	0 and 8
D	6 and 10
E	for no value can A be the domain of f

EXERCISE 15. Find, if possible, the inverse of

$$f(x) = 2x - 3$$

A	$x \mapsto -2x + 3$
B	$x \mapsto 3x - 2$
C	$x \mapsto \frac{2}{3}x - \frac{1}{2}$
D	$x \mapsto \frac{1}{2}x + \frac{3}{2}$
E	none of the preceding