

Lesson 4 - Word Problems

Word problems and equations

In order to solve mathematical **word problems** we often need to use equations. In this lesson, we will learn how to set up *equations* to solve different kinds of word problems.

For example, we will cut up a length of rope into shorter and longer pieces and, given the known total length and other facts, we will calculate the lengths of the pieces cut from it. In other examples we will calculate the ages of two children once we know how many years they are apart and what the sum of their ages is.

We will tackle word problems involving all kinds of numbers: positive and negative integers, including zero; odd and even integers; and consecutive integers.

Two examples of word problems are the following:

Example 1 *Barbara is 2 years older than Silvia. Together the girls are 10 years old. How old are they?*

Example 2 *Six less than a number is 20. Find the number.*

The basic steps

Translating word problems into algebraic equations involves the following basic steps:

1. Read the problem carefully and figure out what it is asking you to find.
2. Assign a *variable* to the quantity you are trying to find.
3. Write down what the *variable* represents.
4. Re-read the problem and write an *equation* for the quantities given in the problem.
5. Solve the equation.
6. Answer the question in the problem.
7. Check your solution.

Let's practice!

Exercise 1 *When 6 is added to four times a number, the result is 50. Find the number.*

Step 1: What are we trying to find?

A number.

Step 2: Assign a variable for the number.

Let's call it n .

Step 3: Write down what the variable represents.

Let $n =$ a number

Step 4: Write an equation.

We are told 6 is added to 4 times a number.

Since n represents the number, four times the number would be $4n$.

If 6 is added to that, we get $6 + 4n$.

We know that answer is 50, so now we have an equation $6 + 4n = 50$

Step 5: Solve the equation. This gives $n = 11$. (Do it as an exercise!)

Step 6: Answer the question in the problem.

The problem asks us to find a number.

We decided that n would be the number, so the number we are looking for is $n = 11$.

Step 7: Check the answer.

The answer makes sense and checks in our equation from Step 4.

Exercise 2 *The sum of a number and 9 is multiplied by -2 and the answer is -8 . Find the number.*

Step 1: What are we trying to find?

A number.

Step 2: Assign a variable for the number.

Let's call it n .

Step 3: Write down what the variable represents.

Let $n =$ a number

Step 4: Write an equation.

We know that we have the sum of a number and 9 which will give us $n + 9$.

We are then told to multiply that by -2 , so we have $-2(n + 9)$.

Be very careful with your parentheses here.

The way this is worded indicates that we find the sum first and then multiply.

We also know the answer is -8 . So we will solve $-2(n + 9) = -8$.

Step 5: Solve the equation. This gives $n = -5$. (Do it as an exercise!)

Step 6: Answer the question in the problem.

The problem asks us to find a number.

We decided that n would be the number, so the number we are looking for is $n = 11$.

Step 7: Check the answer.

Exercise 3 *In a given amount of time, Jamie drove twice as far as Rhonda. Altogether they drove 90 miles. Find the number of miles driven by each by writing an algebraic equation.*

Step 1: What are we trying to find?

The number of miles driven by Jamie and by Rhonda.

Step 2: Assign a variable for the number.

Let's call the number of miles driven by Rhonda x .

Step 3: Write down what the variable represents.

Let $x =$ number of miles driven by Rhonda

We observe that it's not necessary to introduce a new variable to represent the number of miles driven by Jamie. They are in fact simply $2x$.

Step 4: Write an equation.

We know that Jamie and Rhonda drove altogether 90 miles.

So we can write the equation $x + 2x = 90$.

Step 5: Solve the equation. This gives $x = 30$. (Do it as an exercise!)

Step 6: Answer the question in the problem.

The problem asks us to find the number of miles driven by Rhonda and Jamie.

We decided that x would be the number of miles driven by Rhonda, so the answer is $x = 30$.

The number of miles driven by Jamie is $2x$, and this is $x = 2 \cdot 30 = 60$.

Step 7: Check the answer.

More exercises!**Exercise 4** Write each sentence as an algebraic equation.

Sentence	Algebraic Equation
<i>A number increased by nine is fifteen.</i>	
<i>Twice a number is eighteen.</i>	
<i>Four less than a number is twenty.</i>	
<i>A number divided by six is eight.</i>	
<i>A number multiplied by its predecessor is 6.</i>	
<i>A number plus its third part is its half part plus 1.</i>	

Exercise 5 Write each sentence as an algebraic equation.

Sentence	Algebraic Equation
<i>Twice a number, decreased by twenty-nine, is seven.</i>	
<i>Thirty-two is twice a number increased by eight.</i>	
<i>Twelve is sixteen less than four times a number.</i>	
<i>The sum of two consecutive even numbers equals 18.</i>	

Exercise 6 Write each sentence as an algebraic equation.

Sentence	Algebraic Equation
<i>Elen is x years old. In thirteen years she will be twenty-four years old.</i>	
<i>Suzanne made a withdrawal of d dollars from her savings account. Her old balance was \$350, and her new balance is \$280.</i>	
<i>A large pizza pie with 15 slices is shared among p students so that each student's share is 3 slices.</i>	

Geometric word problems

Exercise 7 *A square has an area of sixteen square centimeters. What is the length of each of its sides?*

Exercise 8 *A circle has an area of 49π square units. What is the length of the circle's diameter?*

Exercise 9 *A triangle has a perimeter of 50. If two of its sides are equal and the third side is 5 more than the equal sides, what is the length of the third side?*

Exercise 10 *A rectangle is 4 times as long as it is wide. If the length is increased by 4 inches and the width is decreased by 1 inch, the area will be 60 square inches. What were the dimensions of the original rectangle?*

Exercise 11 *In a quadrilateral two angles are equal. The third angle is equal to the sum of the two equal angles. The fourth angle is 60° less than twice the sum of the other three angles. Find the measures of the angles in the quadrilateral.*